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Techniques for Wide-Angle Distortion Correction Using an Ellipsoidal Projection

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Abstract:

This publication describes techniques of using an ellipsoidal projection for correcting image distortion caused by wide-angle cameras. Two-dimensional (2D) cartesian pixel coordinates of an image taken with a wide-angle camera are first back-projected to three-dimensional (3D) spherical pixel coordinates (*e.g.*, onto a surface of a 3D sphere) using a focal length of the camera. The 3D spherical pixel coordinates are then projected to 3D ellipsoidal pixel coordinates (*e.g.*, onto a surface of a 3D ellipsoid). Major and minor axes of the 3D ellipsoidal pixel coordinates are defined according to a determined face orientation within the image. The 3D ellipsoidal pixel coordinates are then projected to 2D cartesian pixel coordinates to arrive at a distortion-corrected 2D image.

Keywords:

camera, image, wide-angle, selfie, distortion, warping, morphing, processing, correction, projection, ellipsoidal, ellipsoid, mapping, re-project, back-project, 2D, 3D, axes, major, minor, face, orientation, spatially-variant, conformal

Background:

Wide-angle cameras/lenses are being implemented within many imaging devices (*e.g.*, smart-phones, action cameras, digital cameras, security cameras). Although wide-angle cameras produce a wide field-of-view that is desirable, the images taken with such cameras often have portions that are distorted when viewed in two dimensions. For example, when taking a selfie, distortion away from a center of a frame may occur, as shown by the arrows in Figure 1 below.

Figure 1: Distortions away from a center of a frame.

Conventional approaches to correcting such distortion rely on local or global stereographic projections (spherical or cylindrical). While these techniques work for some situations, in some aspects, they do not account for the spatial orientation of the subject matter or, in the case of cylindrical projections, leave one dimension uncorrected, and thus, may not keep perspective projection. As such, it is desirable to have a distortion correction technique that corrects distortions in multiple dimensions while controlling a strength of perspective effect by accounting for the spatial orientation of the subject matter.

Description:

<u>Step 1: Back-project coordinates from a 2D image plane to a 3D sphere</u>

First, an image is captured with pixels having 2D Cartesian pixel coordinates (x, y) away from an origin or center of the image that is at $(0, 0)$. The 2D Cartesian pixel coordinates may be unit coordinates that correspond to a 2D image plane. Adding a focal length parameter, the 2D pixel coordinates become (x, y, f) where f is the focal length. In general, all of the pixels are going to share a focal length, however, if they do not, separate focal lengths may be used for individual pixels or groups of the pixels. In order to back-project the pixel coordinates from the 2D Cartesian pixel coordinates to the 3D spherical coordinates (θ, ϕ), Equation 1 is used.

$$[\theta, \phi] = (cos^{-1}(f/r), tan^{-1}(y/x)), \text{where } r = \sqrt{x^2 + y^2 + f^2} \quad (1)$$

Equation 1 is a stereographic back projection based on a 3D sphere. Effectively, the coordinates of the 2D image space are mapped onto a 3D sphere, for example, as shown in Figure 2 below.

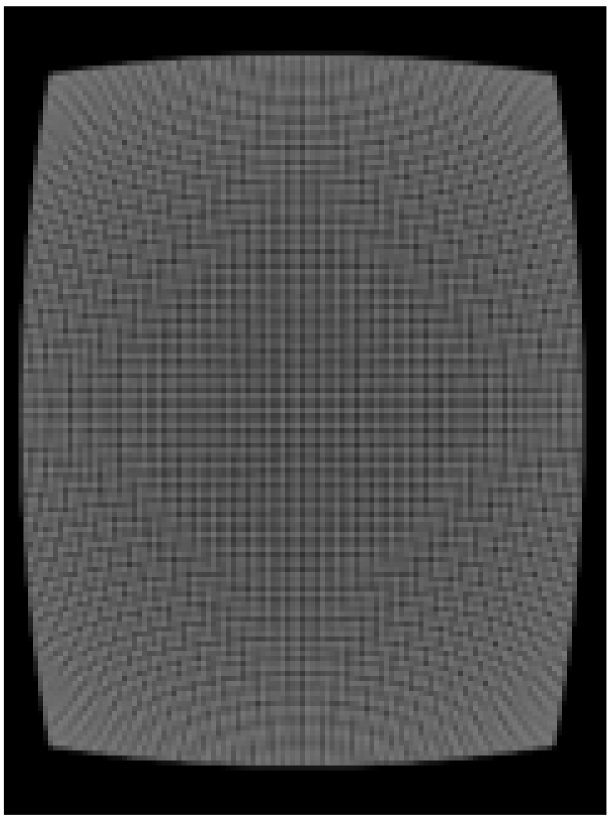


Figure 2: Example spherical projection.

Step 2: Project coordinates from the 3D sphere to a 3D ellipsoid.

Next, the 3D spherical coordinates (θ, ϕ) are mapped from the 3D sphere to a 3D ellipsoid having Equation 2.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{f^2} = 1 \tag{2}$$

In Equation 2, a and b are major and minor axes, respectively. Figure 3 shows a representative ellipsoidal projection where b=2.5f.

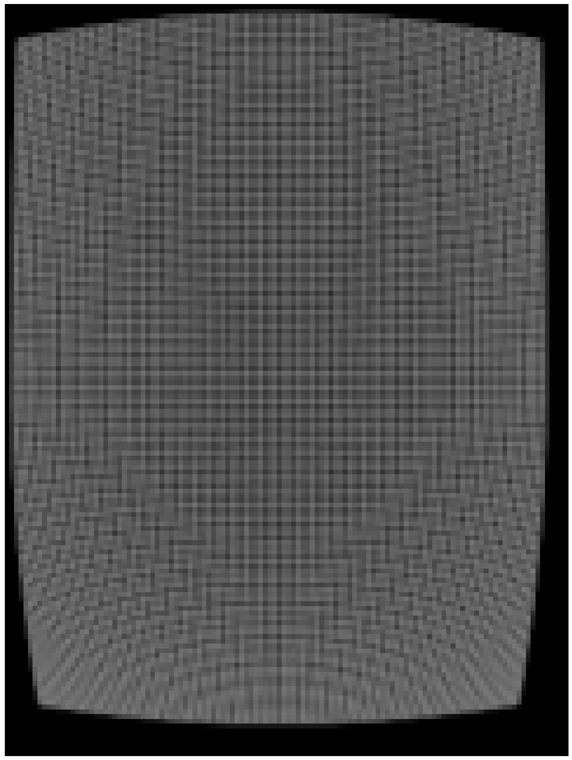


Figure 3: Example ellipsoidal projection.

Based on Equations 1 and 2, the 3D ellipsoidal coordinates on the 3D ellipsoid are given by Equations 3 and 4.

$$u = [\, sin(\theta)cos(\phi), sin(\theta)sin(\phi), cos(\theta) \,] \tag{3}$$

$$r_e{}^2 = 1/((sin(\theta)cos(\phi)/a)^2 + (sin(\theta)sin(\phi)/b)^2 + (cos(\theta)/f)^2) \tag{4}$$

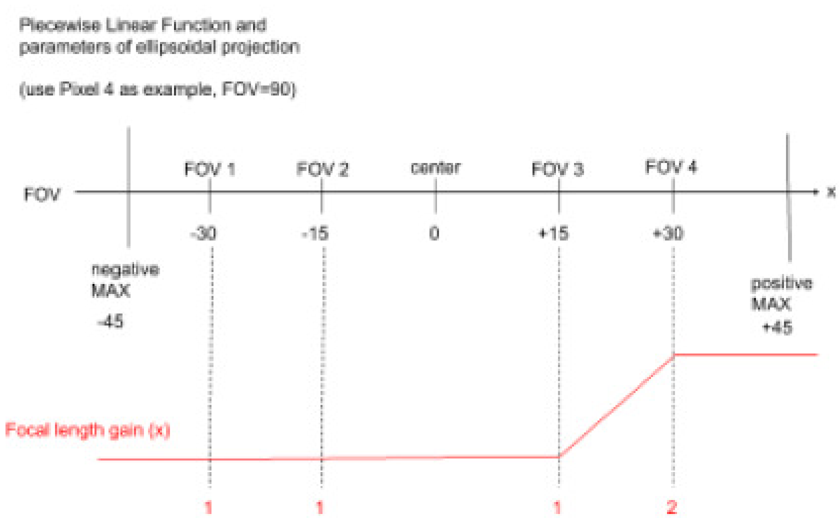
By controlling a and b (major and minor axes), spatially-variant distortion control can be achieved. In order to tune the distortion control, parameters *(a,b)* may be selected based on an orientation of a face detected in the image, *e.g.*, through the use of a face detection algorithm. This allows for perspective projection that is adapted for human faces.

As discussed above, the parameters *(a,b)* allow for tuning of the distortion correction independently in two dimensions:

- When $a = b = f$, the projection becomes a simple stereographic projection.
- When $a/b > f$, the projection uses a longer focal length for x/y direction. This leads to more perspective being preserved in that direction.
- We can further separate a according to a quadrant of (x,y). By doing so, Equation 4 becomes:

  $$r_e{}^2 = 1/((sin(\theta)cos(\phi)/a')^2 + (sin(\theta)sin(\phi)/b)^2 + (cos(\theta)/c)^2)$$

  where $a' = u_x > 0 \;?\; a^+ : a^-$
- The major and minor axes can also be controlled with a piecewise linear curve, as shown in Figure 4 below, to further adjust the corrections in various parts of the image.



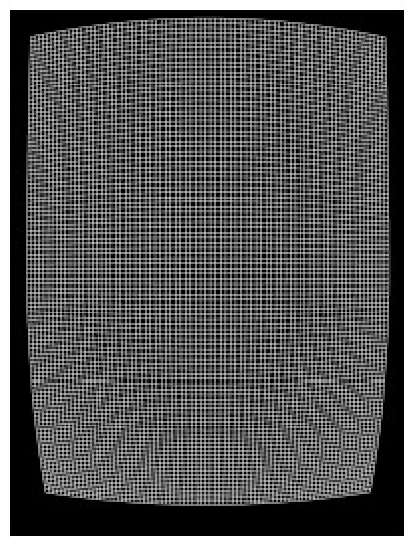


Figure 4: Piecewise axis tuning curve and resultant ellipsoidal projection.

Step 3: Project coordinates from the 3D ellipsoid to the 2D image plane.

The corrected pixel coordinates on the 2D image plane are given by Equation 5.

$$[x', y'] = [r_e u_x, r_e u_y] * 2/(1 + r_e u_z/f) \quad (5)$$

By using Equations 1, 3, 4, and 5, a mapping between the original 2D pixel coordinates $[x, y]$ and the distortion corrected 2D pixel coordinates $[x', y']$ may be generated.

Results:

In all the example figures below, the horizontal skew is corrected, while preserving the vertical perspective effect. For example, the ellipsoidal projections below use an ellipsoid where *b*=2.5*f*

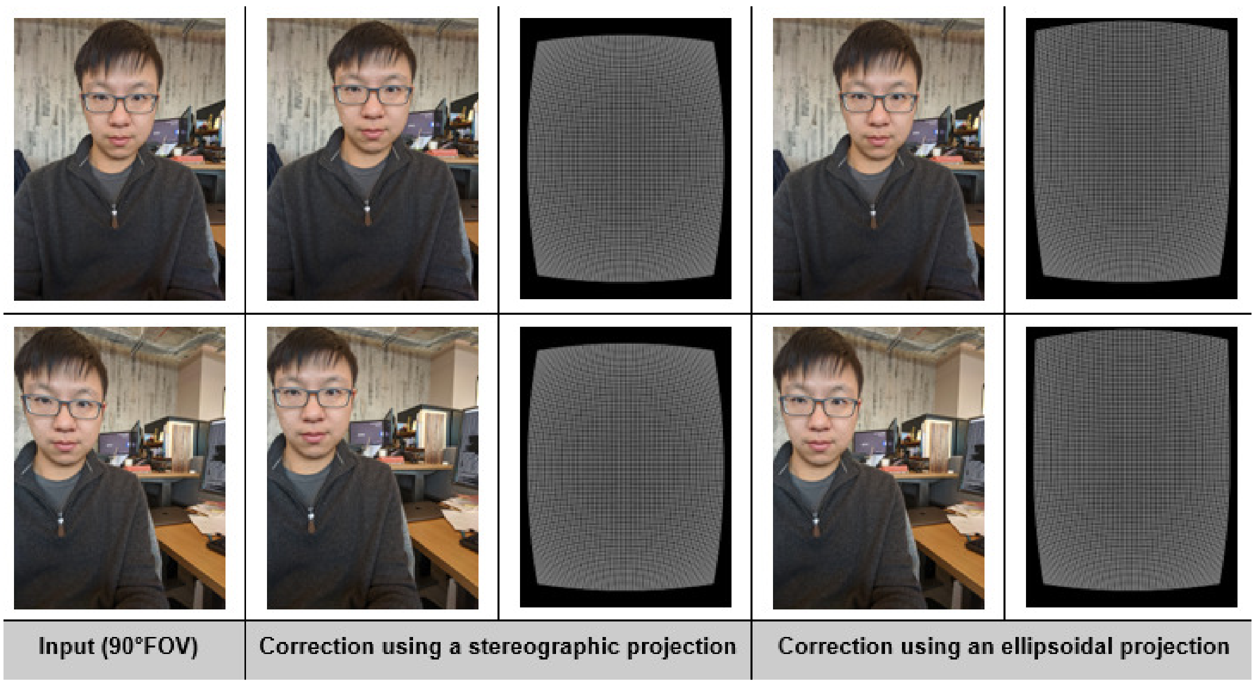


Figure 5: Uncorrected images, images with distortion correction using a stereographic projection, and images with distortion correction using an ellipsoidal projection.

Input (84°FOV)	Correction using stereographic projection	Correction using an ellipsoidal projection

Figure 6: Uncorrected image, an image with distortion correction using a stereographic projection, and an image with distortion correction using an ellipsoidal projection.

Note that the faces in the corrections using the ellipsoidal projections look thinner than those of the corrections using the stereographic projection.

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